Space Elevator Feasibility

What goes up must come down, or does it?

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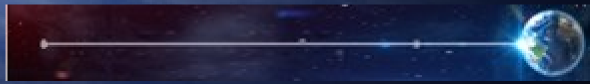
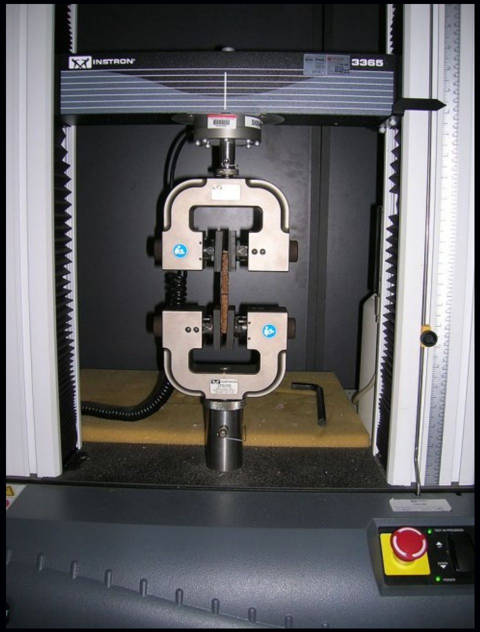


$$1\text{MYuri} := 1\frac{\text{N}}{\text{tex}} = 1\frac{\text{GPa}}{\frac{\text{g}}{\text{cm}^3}} = 1\frac{\text{MNm}}{\text{kg}}$$

Time, Power, Money, and What?

How to put a space elevator aloft?



Tension Tester, Wikipedia

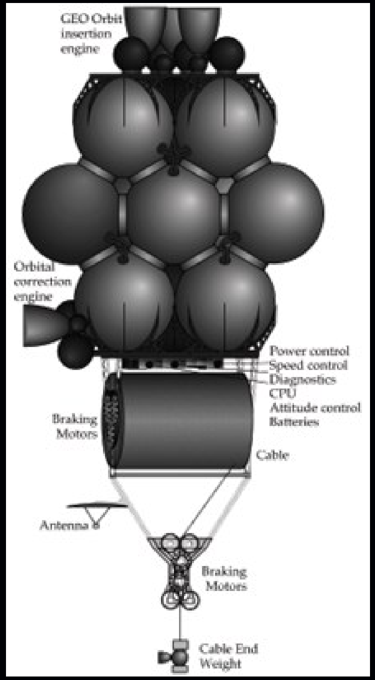
Assumptions:

- A minimal Space Elevator needs to be deployed and cannot be built from scratch from the ground or top. Holds, if tether cannot be produced/added from the balance point under elevator grade tension without already existing tether (Spiderfab/TU?).
- Equal stress formula, exponential taper.

Issues:

1. Can a Space Elevator be constructed/deployed?
   => **Tether, specific tensile strength**

   - Sufficient, that it can carry the tether's weight over the length of the elevator with a small enough total tether mass so that a minimal elevator can be deployed.
   - Works at deployment, fresh material, steady state.
   - No hard limits, 25 MYuri often quoted as lower limit: $\sigma(t_{deploy}) > \sigma_{min} = 25\text{MYuri}$
   - A sufficiently strong tether does not exist yet.



Deployment Spacecraft, http://www.mill-creek-systems.com/HighLift/images/fig3_1.gif

How to keep a space elevator aloft?

2. How does a Space Elevator last?
   => **Tether decay must be balanced with growth/reconstruction**
   (compare first SEFC, Shelef, e.g., presentation, SEC, 2009)

- Works over time, steady state: $\sigma(t_{deploy}) > \sigma(t) > \sigma_{min} = 25\text{MYuri}$
- No catastrophic events
- **Exponential decay of strength with decay time constant t_{SE}:** $\sigma(t) = \sigma(t_{deploy}) e^{-\frac{t}{t_{SE}}} > \sigma_{min}$
- **Replacement of worn tether as payload throughput**

  $t_{\text{lift m\_tether as payload}}$ = $\mathbf{t_{TetherPayload}}$ = $m_{tether}/m_{SEPayload}$ years with $m_{SEPayload}$ /year=(n* $m_{ClimberPayload}$)/year.
- Keep aloft condition: $t_{TetherPayload} < t_{decay\_\sigma min}$ with $t_{decay\_\sigma min} = t_{SE} \ln\left(\frac{\sigma_{deploy}}{\sigma_{min}}\right)$
- Example:

  t_{SE} = 5 years
  σ_{deploy} =50 MYuri (deployment strength)
  σ_{min} =25 MYuri (design strength, taper, etc.)
  => $t_{decay\_\sigma min}$ = 3.5 years
  assumptions: 1 climber/day (with solar power), 6000 mt tether (example)
  => 3.5*52*7 = 3.5*364=1274 climbers => 4.7 mt/climber for tether replacement, required.
  => leaving ~10 mt in a 20 mt climber with 5 mt drive for growth and a doubling time of half the 3.5 years.

Space Elevator Feasibility Condition
(SEFC, existing definitions)

$$t_{TetherPayload} < t_{decay\_\sigma min}$$

- Space Elevator Feasibility Condition (SEFC): $\frac{PMT}{TMR} = \frac{1}{CTC} > \frac{1+FS}{TD} + \frac{1}{TL}$ $\quad \frac{n_{ClimberYear}\, c_{ClimberPayload}}{n_{TMR}} > \frac{1}{t_{decay\_\sigma min}} + \text{growth}$

- Tether
  - Standard Mass Unit (SMU) = m_{std} = m_{max} maximum appendable mass at ground level
  - Tether Mass Ratio (TMR) = m_{tether} / m_{max} = n_{TMR} depends on tether
    safety factor $c_{TetherSafety}$>1 and
    design factor $c_{TetherDesign}$>1,
    let us use TSL = $\sigma(t_{deploy})\, c_{TetherSafety}\, c_{TetherDesign}$
    for calculations of taper and therefore m_{tether} and TMR.

- Power/Climbers
  - Payload Standard Throughput Unit (STU) = m_{max}/year
  - Payload Mass Ratio (PMR) = $m_{ClimberPayload}$ / $m_{ClimberTotal}$ = $m_{ClimberPayload}$ / m_{max} = $c_{ClimberPayload}$<1
  - Payload Mass Throughput (PMT) = $m_{SEPayload}$ /year in STU mit $m_{SEPayload}$ = $n_{ClimberYear}\, m_{ClimberPayload}$
    = $n_{ClimberYear}\, m_{ClimberPayload}$ / m_{max} = $n_{ClimberYear}\, c_{ClimberPayload}$
  - Specific Power (SP) = kW/kg (of power system/infrastructure part of the climber)

- Time
  - Critical Time Constant (CTC) = TMR/PMT years = n_{TMR} / ($n_{ClimberYear}\, c_{ClimberPayload}$) years , larger CTC means lower throughput
  - Normalized Throughput (NT) = 1/CTC = ($n_{ClimberYear}\, c_{ClimberPayload}$) / n_{TMR} /year
    yearly payload throughput normalized to total tether mass

How to grow the space elevator?

- Qualifiers:
  - For growth time equal to replacement time, a factor q_{growth} =2 is inserted:

$$t_{TetherPayload}\, q_{growth} < t_{decay\_\sigma min}$$

  - q_{growth} > 1 is the growth factor for payload in addition to balancing decay. For growth $t_{TetherPayload}$ has to be shortened by q_{growth} or the throughput as the reciprocal value be scaled up accordingly. Doubling time is $t_{decay\_\sigma min}$/ (q_{growth} -1)
  - q_{spare} > 1 is an additional factor to build reserves in orbit to recover from catastrophic failure, just as in the original SEFC).

$$t_{TetherPayload}\, q_{growth} q_{spare} < t_{decay\_\sigma min}$$

  - Example: q_{growth} = 4 (doubling in $t_{decay\_\sigma min}$ 1/3), spare factor q_{spare}=1.25

=> $\mathbf{t_{TetherPayload}} < t_{decay\_\sigma min}/(\, q_g\, q_{spare})=$ $\mathbf{t_{decay\_\sigma min}/5}$

  - If we want to use the SE in addition to these housekeeping tasks, there is another q factor, q_{SEuse}. As before. 50% normal use as compared to housekeeping requires a factor 2 in throughput.

$$t_{TetherPayload}\, q_{growth} q_{spare} q_{SEuse} < t_{decay\_\sigma min}$$

=> $t_{TetherPayload} < t_{decay\_\sigma min}/10$

… or approx. 4 months!

(max of 1800 mt with 1 climber/day and 15mt payload)

$t_{TetherPayload}$ and Climber Power Systems

- Assumptions for Power System:
  - Simplification of the SEFC: Climber divided into payload and structure/power system, e.g., 5 mt of a 20 mt climber with 15 mt payload. That means the climber structure is also in the budget for the power system and the usually quoted numbers for motors alone are not sufficient.
  - Climber structure/power system needs to provide drive to ride the elevator at ~200-300km/h (5d to GEO@300km/h, 7d@200km/h).
- Adopting optimistic function for $t_{TetherPayload}$ (from table on right):
  - Power density for structure and motors
  - 30 MYuri tether
  - 7 months for 3.5kW/kg, 5 mt => 17.5 MW? Radiation cooled?
  - 8.6 months for 1.5kW/kg, 5 mt => 7.5 MW
  - 10 months for 1.0kW/kg, 5 mt => 5 MW
  - 17 months for 0.7kW/kg, 5 mt => 3.5 MW

	TMR Tether mass ratio	A/A_0 Taper ratio	TSL Tether specific loading MYuri[i]						
optimistic	50	2.6	50	2.0	2.7	3.4	4.0	4.6	5.0
	77	3.4	40	1.3	1.8	2.2	2.6	3.0	3.2
CNT	144	5.0	30	.69	.94	1.2	1.4	1.6	1.7
CNT	228	7.0	25	.44	.59	.75	.88	1.0	1.1
	433	11.3	20	.23	.31	.39	.46	.53	.58
pessimistic	739	17.3	17	.14	.18	.23	.27	.31	.34
SP Specific power kWatt/kg				0.5	0.7	1.0	1.5	2.5	3.5
PMT Optimized payload throughput STU				100	135	170	200	230	250
				pessimistic		Thin-film receiver + motors			optimistic

For example:
A space elevator constructed with a 30 MYuri tether and 3.5 kWatt/kg motors, can lift its own mass 1.7 times a year.

Blue values are optimistic, Red values are pessimistic, gray values are probable. As expected, stronger tethers and lighter power systems influence the normalized throughput in a similar manner and so are interchangeable.

Table 2: Tether vs. power driven constraints and their effect on the possible normalized throughput: $NT_{pos} = 1/CTC_{pos} = PMT/TMR$

But can it be climbed?

3. Can the Space Elevator be climbed?

   => **Tether, minimal friction**

- Example Scenario:
  - 40 GPa tether (no safety factor, just to get to simple numbers, approximately 30 MYuri)
  - Carries a 20 mt climber

  => Tether cross section area?

$$\frac{200\text{kN}}{40\text{GPa}} = \frac{1}{200000}\text{m}^2 = 5\text{mm}^2$$

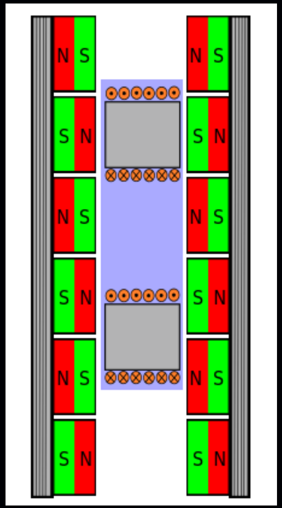
5 mm^2 tether cross section area (0.008 in^2, 10-11 AWG wire)

  - Tether Shape can vary from
    - Round (1) $r^2\pi$
      - ~ 2.5 mm diameter
      - ~ 80 cm^2 (12 in^2) surface for 1 m tether length
    - to Flat $d \cdot w$
      - Ribbon with the Following Examples:
        - 5 µm thick, 1 m wide => ~ 20000 cm^2 surface per 1 m tether
        - 10 µm thick, .5 m wide => ~10000 cm^2 surface per 1 m tether (2) (125x)
        - 20 µm thick, .25 m wide => ~5000 cm^2 surface per 1 m tether
        - .5 mm, 1 cm wide => 210 cm^2 (3) (2.6x)

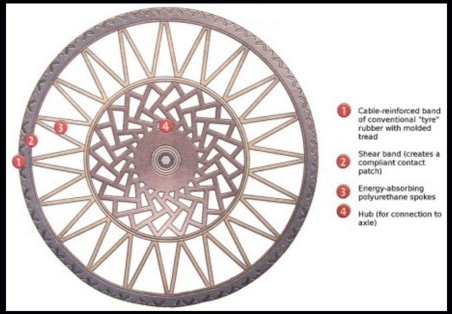
Comparison:

Sheet of 80g bond paper ~ 100 µm thick

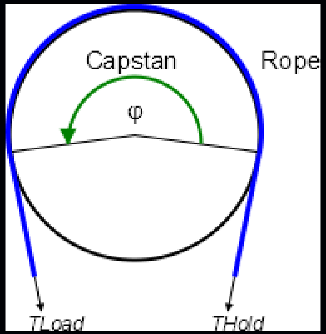
Many Categories of Climber-Tether Interfaces Proposed (Examples)

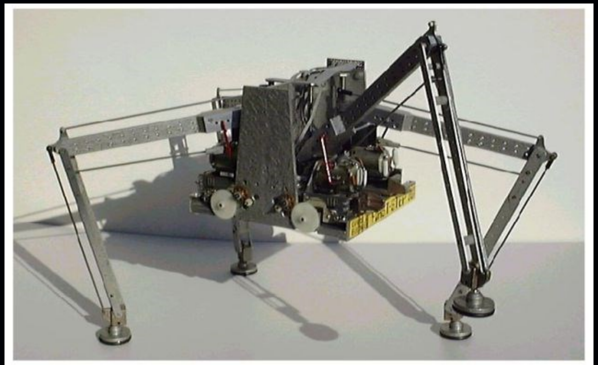


- Contactless
  (Fields Instead of Mechanical Touch Points)
  - Linear Motor (Maglev)



- Mechanical Contact
  - Wheels
    - Capstan
      (tether is wrapped around solid wheel to increase contact area)
    - Pinching Wheel
      (flexible wheel is deformed to increase contact area)
    - Tracks
    - Hybrids
  - No Wheels
    - Walker
    - Inchworm
    - Tether-vibration Drive





Climber-Tether Interaction

- Forces: friction f, weight $w_{climber}(h)$, motor torque, inertial moments have to balance
  - Friction: $f = \mu \frac{F}{m^2} a$ material coefficient · pressure · contact area

    a=contact area (it is a complex issue), μ=friction coefficient, F=force normal to tether, unit area in m^2

    -> f = friction force

    1. f is essential to balance weight AND acceleration of a climber
    2. High strength material for a SE => tiny mass and surface => good f is a challenge!

  - Weight of Climber: $w_{climber}(h) = m_{climber} \cdot g(h), g(h) = \frac{m_{Earth} G}{(r_{Earth} + h)^2} - (r_{Earth} + h)\omega^2$

  - Motor Torque -> Climber Acceleration: $F_{acc} = \frac{60 \frac{m}{s}}{100\ s} \cdot 20000\ kg = 12\ kN\ (adds5\%)$

Condition for f: $f > w(h) + F_{acc} + residuals$

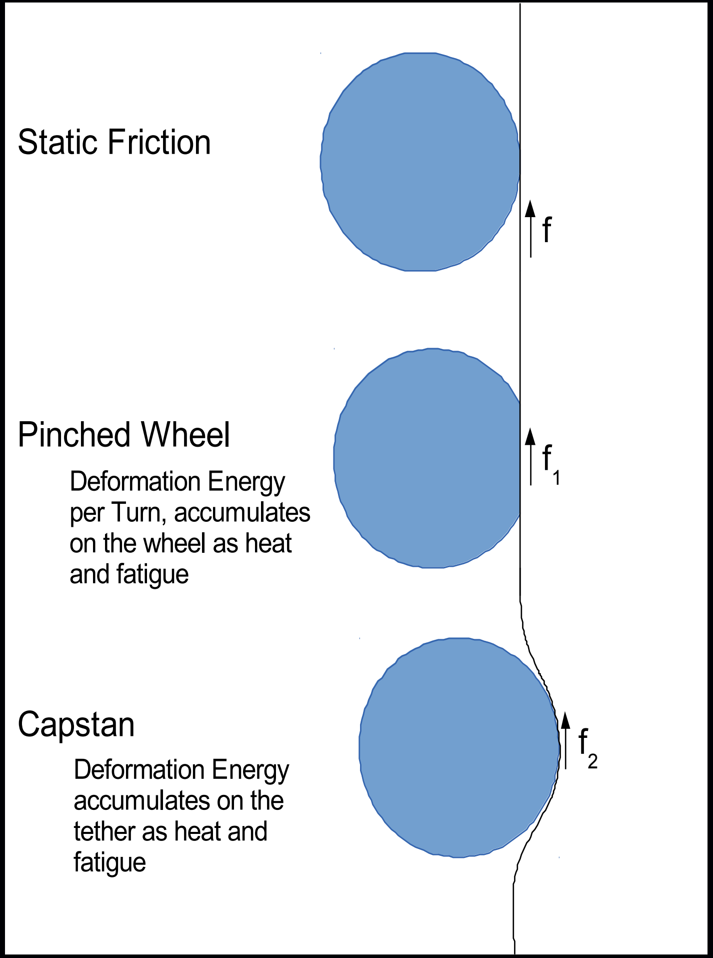
Friction, Heat, and Fatigue I

- Friction is a complex systemic phenomenon:
  - Coulomb approximation

    dry, atomically close contact on small fraction of surface, contact area is proportional to normal force until saturation

    Not exactly linear
  - Adhesive tape is different

    no force, proportional to area -> Drag Racing Tires
  - Gecko/Nanotubes -> active control of adhesion?
  - Nonelastic Processes (Internal Friction) Result in Heat and Mechanical Wear/Fatigue of Wheels and Tether
- Maximizing Friction by Increasing the Active Surface: Wheel Size, Pinched Wheel, Capstan (climbers in competitions often use a hybrid solution)
- Heat:
  - Only radiation-cooling of the climber
  - Heat created from deformation of pinched wheel per rotation
  - Heat left on the tether has more time to radiate/dissipate
  - But heat deposited at a spot is quickly destroying the tether, e.g., slipping wheels.
- Fatigue
  - Inelastic wear per wheel rotation. How much repeated pinching, bending, and stretching (capstan) can a 10 µm tether tolerate without losing integrity? (scotch tape backing: 38 µm, packing tape: 80 µm)
  - Exchange/loss of materials between climber and tether



Heat and Fatigue appear possible to be adressed with proper choice of wheel size (limiting the number of turns)

Friction, Heat, and Fatigue II

- Example for Space Elevator
  - Wheel rotations to GEO: $\frac{11.4 \cdot 10^6}{d_{wheel}}$ d in meters
  - Friction Coefficients:
    - High: Brake Material/Cast Iron: 0.4, graphite-graphite in vacuum: 0.8, Low: Carbon/Carbon: 0.1
    - Force is transfered to tether over what length?
    - Size of contact patch required for 20 metric tons:
      - 1000 N/cm^2 -> 10 MPa (100 kg/cm^2 , 1500psi)
        200 cm^2 contact patch with friction coefficient 1 (0.02 m^2)
      - Tether length required for contact: (1) 2.5 m, (2) 0.02 m, (3) 0.95 m
      - Lower friction µ than 1 extends req. contact by factor of 1/µ => with 0.25 assumed for CNTs: (1) 10 m, (2) 0.08 m, (3) 3.8 m
        How many wheels to reach this? In 5 mt payload budget?
      - Wheels extend length substantially by the factor of wheel diameter to contact patch length.
      - Thin tether reduces contact patch and points to pinched wheels and high stress on the tether.
  - Fatigue:
    - Rough wheel coating as used in competitions might puncture a SE tether
    - Tradeoff between contact area/number of wheels and number of revolutions to reach GEO

Reality Check

⇒**Geometry Condition:**

- Minimal Tether thickness: 0.5mm
- Round tether preferred
- Minimal surface to work (provide enough friction) with $\mu=0.5$

- **Overrides Tether strength!**
- Only a large enough elevator can be climbed and maintained!

- HHenriksen Atlas APA-5 Power Ascender
  - Lifts 300kg
  - 7/16” (11mm) rope (elasticity and capstaning)

- Comparison: Required for Space Elevator, 20 mt climber, 30 MYuri tether
  - 70x more lifted mass
  - 76x smaller cross section area of tether
  - with no capstaning or much elasticity in the tether

4. Economy

- SpaceX is the benchmark to beat
  - Quoted cost for Falcon Heavy is
    - USD 1400/kg for LEO
    - USD 3370/kg for GTO
    - USD 5360/kg for Mars
  - Reusable rockets offer other modes of mobility
  - Cryo-Propellant is not cheap
- Elevator at USD 20B development and construction costs needs to lift ~4 M kg to break even for the Mars target.

Summary

- A simplified and extended feasibility condition for Space Elevators was introduced. Time balance for tether and climber power, and a new geometry condition so the elevator can be climbed.
  - Time/Tether/Power Condition

$$t_{TetherPayload}\, q_{growth} q_{spare} q_{SEuse} < t_{decay\_\sigma min}$$

  - Geometry Condition:
    - Minimal Tether thickness: 0.5mm (TBD)
    - Minimal surface to work (provide enough friction) with μ=0.5
    - Round tether preferred
- Attention for SE feasibility needs to be first on a suitable material.
  - This is useful for many other space applications, even if during development not yet Space Elevator grade.
    - Corollary: Tensile Structures are useful in Space, from EM-Tethers such as EDDE (Pearson at al) and MXER (Tethers Unlimited), to Solar- and Magnetic Sails, to concepts such as Space Elevators and Valkyrie (Pellegrino).
  - 25 MYuri may get an elevator up, but to keep it up a higher Myuri number and decay need to be factored in.
- Other basics such as friction should not be ignored. It is as challenging as material strength. A stronger material leaves less to hold on to.
- In housekeeping operations, decayed, lower grade space elevator material may need to be removed to reduce parasitic mass, while new material is added. Both at > 200km/h.

- Space Elevators remain challenging.

Space Elevator Cable … Still Priceless

Thank You!